

Problem Set 9

Problem 1

Let X_1, \dots, X_n be independent random variables with density functions f_1, \dots, f_n . Furthermore, let $f_i(x) \leq \phi$ for every $i \in \{1, \dots, n\}$ and every $x \in \mathbb{R}$. Give an upper bound for the probability of $X_1 + X_2 + \dots + X_n \in [a, a + \varepsilon]$ where $a \in \mathbb{R}$ and $\varepsilon > 0$ are fixed arbitrarily. [Hint: Think of Lemma 6.6!]

Can you extend your reasoning to bound the probability of $\lambda_1 X_1 + \dots + \lambda_n X_n \in [a, a + \varepsilon]$, where $\lambda_1, \dots, \lambda_n \in \mathbb{R}_+$ are arbitrary numbers?

Problem 2

We want to get more familiar with the notion of ϕ -perturbed values. Assume that c is drawn independently from a distribution with density function $f(x)$, where

1. $f(x)$ is the density of the uniform distribution over the interval $[4, 4 + u]$ for a constant $u > 0$.
2. $f(x) = \begin{cases} x^2 \cdot \frac{3}{u^3} & \text{for } x \in [0, u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (0, \infty)$.
3. $f(x) = \begin{cases} \frac{1}{x} \cdot \frac{1}{\ln u} & \text{for } x \in [1, u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (1, \infty)$.

For all three cases, do the following:

- a) Compute (a best possible) ϕ such that $f(x) \leq \phi$ for all $x \in \mathbb{R}$.
- b) Give a best possible upper bound $\nu(u, \epsilon)$ for the probability to draw a number from a given fixed interval of width $\epsilon \in (0, 1)$.
- c) Assume someone tells us that c is 3-perturbed. Based on this fixed $\phi = 3$, for which of the three scenarios do we get the largest (i.e. worst) $\nu(u, \epsilon)$?