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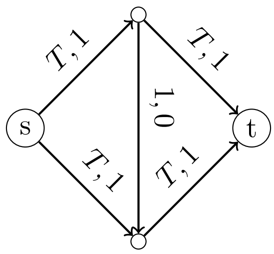
Randomized Algorithms and
Probabilistic Analysis
Summer 2017

Problem Set 12

Problem 1

Recall the following instance from problem set 11.



1. Give a short proof that changing the costs of the five edges by choosing them according to independent density functions $f_e : [0,1] \to [0,\phi]$ implies that the SSP algorithm converges in a constant number of steps for any integer T. Assume that Property 8.9 from the lecture is proven and use it.
2. Give an even shorter proof that uses Property 8.9 and Corollary 8.3.
3. Extend your proof from 2. to arbitrary input graphs of constant size.

Problem 2

Let $G = (V, E)$ be a graph let $m = |E|$ be the number of edges. Let $\mathcal{S}$ be the family of all feasible matchings in G. For this task, it is irrelevant what the matching property is, it is only important that $\mathcal{S}$ is a family of subsets of E.

Let L be some positive integer. Assume that we draw a weight $w(e)$ for each edge independently and uniformly at random from $\{1, \ldots, L\}$. The weight of a set $M \in \mathcal{S}$ (a matching) is defined as $w(M) = \sum_{e \in M} w(e)$.

Let M^* be an element from $\mathcal{S}$ with maximum weight (a maximum matching), i.e. $w(M) \leq M^*$ for all $M \in \mathcal{S}$. Prove that the probability that M^* is the *unique* element in $\mathcal{S}$ with weight $w(M^*)$ is at least $1 - \frac{m}{L}$. In other words, prove

$$\Pr(\exists M' \in \mathcal{S} \backslash M^* : w(M') = w(M^*)) < \frac{m}{L}.$$