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Due date: In the lecture on 10th July 2019, 12:15

Algorithmic Game Theory and the Internet

Summer Term 2019

Exercise Set 13

Exercise 1: (4 Points)
Show that there are instances of the stable matching problem in which the Gale-Shapley Algorithm (men-proposing algorithm) runs for $\Omega(n^2)$ iterations before terminating (with a stable matching). For this purpose, state an instance of the problem depending on n with suitable chosen preference orders and lower bound the number of iterations of the algorithm.

Hint: Consider an instance with $|U| = |V| = n$. Try to enforce exactly one rejection per iteration.

Exercise 2: (3 Points)
Prove that the men-proposal algorithm is not DSIC for the right-hand side (the women). For this purpose, give an instance of the stable matching problem in which, by lying about her preferences during the execution of the men-proposing algorithm, a woman can end up with a man that she prefers over the man she would have ended up with had she told the truth.

Exercise 3: (2 Points)
Show that there is not necessarily a solution to the stable roommates problem: In this problem, there is a set of $2n$ people, each with a total preference order over all the remaining people. A matching of the people (each matched pair will become roommates) is stable if there is no pair of people that are not matched that prefer to be roommates with each other over their assigned roommate in the matching.

Exercises 4 and 5 on the next page.

Exercise 4: (2+2 Points)

We want to deepen our understanding of cake cutting protocols. For this purpose, consider the following subtasks:

(a) The famous cake cutting protocol *cut and choose* for two agents can also be considered as a direct mechanism: Both agents simultaneously report a valuation density function $b_i \colon [0,1] \to \mathbb{R}_{\geq 0}$ for $i = 1, 2$. Afterwards, the mechanism cuts the point t such that $\int_0^t b_1(x)dx = \int_t^1 b_1(x)dx = \frac{1}{2}$. Then it allocates that piece of $[0, t)$ and $[t, 1]$ to agent 2 that maximizes her declared value leaving the other piece for agent 1.

Prove that the given mechanism is not DSIC.

(b) Consider the algorithm (which is also known as the *moving-knife algorithm*) given in Section 4 of Lecture 23 that determines a proportional allocation for any number of agents n.

Show that even in the case of three agents the allocation of the algorithm might not be envy-free.

Exercise 5: (3+4 Points)

The *participation criterion* requires that the addition of a voter who strictly prefers candidate A to B should not change the winner from candidate A to candidate B.

(a) Prove that plurality voting satisfies the participation criterion.

(b) Show that instant runoff voting does not satisfy the participation criterion.

Hint: It's possible to solve subtask (b) in two steps: First, state a counterexample in which you add a couple of voters in order to show that the property is violated. Then, you can think of sequentially adding those voters to the original instance and detecting the point in which the winner changes from candidate A to B.