Institut für Informatik
Prof. Dr. Heiko Röglin
Dr. Daniel Schmidt

UNIVERSITÄT BONN

Randomized Algorithms and
Probabilistic Analysis
Summer 2020

Problem Set 11

Please hand in your solutions via e-mail (ipsarros@uni-bonn.de) until Monday July 6th.

Problem 1

1. Recall Problem 1 from Problem Set 10, where you considered the following *product knapsack problem*. Given n objects with deterministic weights $w_1, \dots, w_n \in [0,1]$, a capacity W and deterministic profits $p_1, \dots, p_n \in \mathbb{R}^{\geq 1}$, find a solution $x \in \{0,1\}^n$ that maximizes the product

   $$p(x) = \prod_{i:x_i=1} p_i$$

   of the profits of the chosen items under the constraint that $w^t x \leq W$. You argued that the Nemhauser-Ullmann algorithm can be adapted to compute an optimal solution to the product knapsack problem. What can you say about the smoothed complexity of the product knapsack problem?

2. Recall Problem 2 from Problem Set 10. Let $G = (V, E)$ be a graph with edge lengths $\ell : E \to [0,1]$ and edge costs $c : E \to [0,1]$. Let $\{s,t\} \in V$. Assume that the edge costs are ϕ-perturbed values. What is the expected number of Pareto-optimal paths?

Problem 2

How can we solve the fractional knapsack problem in linear time? You may assume that the greedy algorithm for fractional knapsack (page 105) is correct, and that there exists an algorithm that finds the median of n numbers in time $\mathcal{O}(n)$.

Problem 3

We discuss bad examples for the core algorithm. In the following, you may use fractional weights and profits.

1. The idea behind the core algorithm is the hope that the optimal solution for the fractional knapsack problem as computed by the greedy algorithm on page 105, $\bar{x}$, typically differs from an optimal solution in only in a few items, and that these items lie in the core. Give an example where, except for the break item, no item that is in $\bar{x}$ is contained in any optimal solution x^*.

2. The expanding core algorithm estimates the additive integrality gap Γ by iteratively increasing it. Give an example where it increases the estimator $n-1$ times and finally ends up with all items in the core.