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Due: July 15, 2020 at noon

Algorithms and Uncertainty

Summer Term 2020

Exercise Set 10

Exercise 1: (2+2+2+3 Points)

For the normed vector space $(\mathbb{R}^d, \|.\|)$, the unit ball with respect to $\|.\|$ is defined as the set $\{x \in \mathbb{R}^d : \|x\| \leq 1\}$.

(a) Show that the unit ball with respect to the 1-norm is convex.

(b) Show that the unit ball with respect to the 2-norm is convex.

(c) Show that the unit ball with respect to the ∞-norm is convex.

(d) Does the same result hold for an arbitrary p-norm with $p > 1$? What about $p < 1$?

Exercise 2: (3 Points)

Prove Observation 21.4: If R is σ-strongly convex and $f_1, f_2, \ldots$ are convex then $R + \sum_t f_t$ is σ-strongly convex.

Exercise 3: (4 Points)

We consider Online Linear Regression as introduced in the lecture. Recall that

$$f_t(w_1, w_2) = \left(w_1 x^{(t)} + w_2 - y^{(t)}\right)^2 \quad .$$

Derive a regret bound for Follow-the-Regularized-Leader with Euclidean regularization under the assumption that $|x^{(t)}|, |y^{(t)}| \leq 1$ for all t and $S = \{\mathbf{w} \in \mathbb{R}^2 \mid \|\mathbf{w}\|_2 \leq r\}$.

Exercise 4: (4 Points)

Derive a regret bound for Follow-the-Regularized-Leader if the Lipschitz constant depends on the time step, that is,

$$f_t(\mathbf{u}) - f_t(\mathbf{v}) \leq L_t \|\mathbf{u} - \mathbf{v}\| \quad \text{for all } \mathbf{u}, \mathbf{v} \in S \ .$$