Thomas Kesselheim
Alexander Braun

April 22, 2021

Algorithms and Uncertainty

Summer Term 2021

Tutorial Session - Week 2

You are supposed to work on these tasks in class together with your fellow students. Therefore, you are sent into Zoom Breakout-Rooms together with 1-3 other students. Once entered, make sure your camera and microphone are switched on. If you do not know each other yet, each of you could start with a very quick introduction: What's your name? Do you study Computer Science or maybe something else (Maths, Economics,...)? Do you have any prior knowledge on Algorithms and Uncertainty (Online Algorithms, Markov Decision Processes, Learning Problems,...) already or is this your first course in this area?

Afterwards, you are supposed to discuss the exercises on this sheet. Note that you should see this also as a chance to talk about definitions, proof ideas and techniques in addition to only working out a formal solution for the tasks. If you do not know a definition or theorem by hard, feel free to open the lecture notes and have a look. Further, if you have any questions, I will drop by in your Breakout-Room to discuss possible issues with you.

If there is some time remaining at the end of the tutorial, all of us will meet again so that you can share your ideas on the tasks with the whole group.

Exercise 1:
Consider the following Set Cover instance: $U = \{1, 2, 3\}$ and $\mathcal{S} = \{A, B, C\}$ with $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{2, 3\}$, $c_A = c_B = 3$, $c_C = 4$.

(a) Give an optimal integral solution.

(b) Give a fractional primal solution of cost at most 5.

(c) Give a dual solution of value at least 5.

(d) Use your solution of (c) to show optimality of your solution of (b). To this end, sum up the primal constraints in a suitable way. (Your solution should be in the spirit of proof of weak duality but not use the statement of the lemma itself.)

Exercise 2:
Given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the *frequency* of the set system.
Consider the unweighted version of Online Set Cover, i.e., $c_S = 1$ for all $S \in \mathcal{S}$, and the following algorithm: Upon arrival of element e, if $\sum_{S: e \in S} x_S = 0$, set $x_S = 1$ for all S with $e \in S$ and $y_e = 1$. Otherwise set $y_e = 0$. Show that this algorithm is f-competitive by using Lemma 3.7.