Algorithmic Game Theory

Summer Term 2023 Tutorial Session - Week 3

As last week, please find yourself in groups of up to three students. Start with a quick introduction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by hard.

Exercise 1:

- a) Specify the payoff matrix for the well-known game rock-paper-scissors-well. It is a variation of rock-paper-scissors¹ with an additional weapon "well", which beats scissors and rock but looses against paper. Assume that winning has a cost of -1, losing a cost of 1, a tie a cost of 0.
- b) We define a strategy $s_i \in S_i$ of a normal-form cost-minimization game to be *dominated*, if there exists a strategy s'_i such that $c_i(s'_i, s_{-i}) \leq c_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$. Is there a dominated strategy in rock-paper-scissors-well?
- c) Compute a mixed Nash equilibrium. Could you have guessed it?

Exercise 2:

Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE-kSAT) which is defined the following way:

- **Instances:** Propositional logic formula with n binary variables x_1, \ldots, x_n that is described by m clauses c_1, \ldots, c_m . Each clause c_i has a weight w_i and consists of exactly k literals, which are all positive (i.e., the formula does not contain any negated variable \overline{x}_i).
- **Feasible solutions:** Any variable assignment $s \in \{0, 1\}^n$
- **Objective function:** Sum of weights of clauses c_i in which not all literals are mapped to the same value.
- **Neighbourhood:** Assignments s and s' are *neighbouring* if they differ in the assignment of a single variable.

Show that Pos-NAE-kSAT is in PLS.

¹https://en.wikipedia.org/wiki/Rock_paper_scissors