

Algorithmic Game Theory

Summer Term 2023

Tutorial Session - Week 5

As last week, please find yourself in groups of up to three students. Start with a quick introduction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by hard.

Exercise 1:

State for each $M \geq 1$ a network congestion game with two players such that the Price of Anarchy of pure Nash equilibria is at least M .

Exercise 2:

In Lecture 7, we did assume that $\ell_i^{(t)} \in [0, 1]$ for all i and t . We would now like to consider the setting, in which $\ell_i^{(t)} \in [-\rho, \rho]$ for all i and t and some fixed $\rho > 0$. Can you state a no-regret algorithm for this case? Also give a bound for the regret. You should reuse algorithms and results from the lectures.

Exercise 3:

In the lecture we presented the Multiplicative-Weights Algorithm (MW) as an example for a no-external-regret algorithm with an a priori known and fixed time horizon T . Can you state a no-external-regret algorithm which does not need the parameter T ?

Hint: You may want to use the algorithm of the lecture as a subroutine. Initially, assume $T = 1$ and make use of the subroutine. Once a subroutine ends, double the parameter T and restart the subroutine.

Exercise 4:

State an example of a sequence of probability distributions $p^{(t)}$ over strategies and cost vectors $\ell^{(t)}$ such that the player's external regret is negative.