

## Algorithmic Game Theory

Summer Term 2023

Tutorial Session - Week 11

*As last week, please find yourself in groups of up to three students. Start with a quick introduction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by hard.*

### Exercise 1:

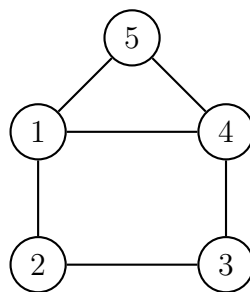
Consider the following instance of the house-allocation problem. There are six agents  $a, \dots, f$  and their preferences are given by:

$$\begin{aligned} a : b > d > f > e > c > a, & \quad b : d > a > c > e > f > b, \\ c : e > f > a > c > b > d, & \quad d : e > a > b > c > d > f, \\ e : f > e > c > b > d > a, & \quad f : d > a > b > c > f > e. \end{aligned}$$

Find a stable allocation  $\pi$  using the Top Trading Cycle Algorithm.

### Exercise 2:

Consider the problem of Pairwise Kidney Exchange by Matching from Lecture 22. The graph below depicts an instance of agents (that is, patient-donor pairs) and possible pairwise exchanges, i.e. nodes represent patient-donor pairs with an edge connecting two nodes if an exchange between the two patient-donor pairs is possible.



Use the mechanism of Section 4 from Lecture 22 and consider agents in ascending order of agent indices (which is independent of the reports) to determine the set of maximum matchings  $M_5$ .