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Due date: June 06, 2023 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2023

Exercise Set 7

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (3 Points)

Recall the auction of k identical items from the previous exercise sets: Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0.

Make use of the VCG-results from the lecture in order to design a truthful mechanism for this auction. For this purpose, explicitly state the function f and calculate the payment rule p.

Exercise 2: (4 Points)

As seen in the lecture, let $f\colon V \to X$ be a function that maximizes declared welfare, i.e., $f(b) \in \arg\max_{x\in X} \sum_i b_i(x)$ for all $b \in V$. For each i, let h_i be an arbitrary function $b_{-i} \mapsto h_i(b_{-i})$ which does not depend on b_i. We define a mechanism $\mathcal{M} = (f, p)$ by setting

$$p_i(b) = h_i(b_{-i}) - \sum_{j\neq i} b_j(f(b)) \ .$$

Prove that $\mathcal{M}$ is a truthful mechanism.

Exercise 3: (4+2 Points)

We consider a single-item auction via a mechanism which follows the spirit from Lecture 14, Section 2: All bidders submit their bids b_i. Fix a price of p (may depend on b) for the item. Approach bidders in order $1, \ldots, n$. As we consider bidder i: if the item is not allocated yet, assign the item for a price of p if $b_i - p \geq 0$.

(a) If $b = v$, show that the social welfare obtained by this auction is at least

$$\max_i v_i \mathbb{1}_{\text{item not allocated}} + p\left(\mathbb{1}_{\text{item allocated}} - \mathbb{1}_{\text{item not allocated}}\right) \ .$$

(b) Use your result from (a) to set a price obtaining a social welfare of at least $\frac{1}{2}\max_i v_i$ if $b = v$.