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Due: lecture on October 23, 2018

Algorithms and Uncertainty

Winter Semester 2018/19

Exercise Set 1

Exercise 1: (2+2+2+2 Points)
Consider the following Set-Cover instance: $U = \{1, 2, 3\}$ and $\mathcal{S} = \{A, B, C\}$ with $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{2, 3\}$, $c_A = c_B = 3$, $c_C = 4$.

(a) Give an optimal integral solution.

(b) Give a fractional primal solution of cost at most 5.

(c) Give a dual solution of value at least 5.

(d) Use your solution of (c) to show optimality of your solution of (b). To this end, sum up the primal constraints in a suitable way. (Your solution should be in the spirit of proof of weak duality but not use the statement of the lemma itself.)

Exercise 2: (3+4 Points)
Given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the *frequency* of the set system.

(a) Consider the unweighted version of Online Set Cover, i.e., $c_S = 1$ for all $S \in \mathcal{S}$, and the following algorithm: Upon arrival of element e, if $\sum_{S: e \in S} x_S = 0$, set $x_S = 1$ for all S with $e \in S$ and $y_e = 1$. Otherwise set $y_e = 0$. Show that this algorithm is f-competitive by using Lemma 2.7.

(b) Now, we generalize the algorithm from (a) to the weighted version. Let $g_e = \max\{0, 1 - \sum_{S: e \in S} x_S\}$ and let S_e be the cheapest set covering e. For each S that covers e, increase x_S by $\frac{c_{S_e}}{c_S} g_e$ and set $y_e = c_{S_e} g_e$. Show that this algorithm is f-competitive by using Lemma 2.7.

Exercise 3: (5 Points)
Again, given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system.
Use our results from the third lecture to devise an online algorithm that is $O(\log f)$-competitive for fractional set cover and prove this. You may assume that f is known beforehand.

Hint: One bound in the analysis from the lecture can be improved for $f < n$. Use it to adapt the algorithm.