MA-INF 1203 Discrete and Computational Geometry

Wintersemester 2019/20
Assignment 11

Deadline: **14** January before noon (To be discussed: **14/15**. January 2020)

1 The well-separated pair decomposition (WSPD)

Consider the points $p_1 = (0.05, 0.01)$, $p_2 = (0.07, 0.01)$, $p_3 = (0.12, 0.15)$, $p_4 = (0.3, 0.3)$, $p_5 = (0.63, 0.68)$, $p_6 = (0.68, 0.68)$ as depicted in Figure 1. Find the WSPD of this point set with separation ratio $s = 3$, which is obtained by the construction algorithm presented in class.

2 Construction of WSPDs

Analyse the running time and the size of the output of the WSPD construction algorithm which uses regular (uncompressed) quadtrees instead of compressed quadtrees (consider a pair of nodes u, v, where one of them is a leaf without any input points, to be not well-separated).

3 Spanners based on WSPDs

Find an exact upper bound on the size of the WSPD for a set of points in $\mathbb{R}^3$ that is constructed by the algorithm if we use a compressed quadtree and separation ratio $s = 4\frac{t+1}{t-1}$, where $t = 2$ (for example, in order to construct a 2-spanner).

4 Approximating the diameter

Use WSPDs to design an algorithm for the problem of approximating the diameter of a point set. Given a set P of n points in $\mathbb{R}^d$ and error tolerance $\epsilon > 0$, the algorithm must return two points $p', q' \in P$ such that

$$\|p' - q'\| \geq \frac{\max_{p,q \in P} \|p - q\|}{1 + \epsilon}.$$

Analyse the running time of your algorithm. You may assume that:

- a compressed quadtree on P can be constructed in $O(n \log n)$ time
- one can decide in $O(1)$ time whether any pair of canonical cubes is well-separated.

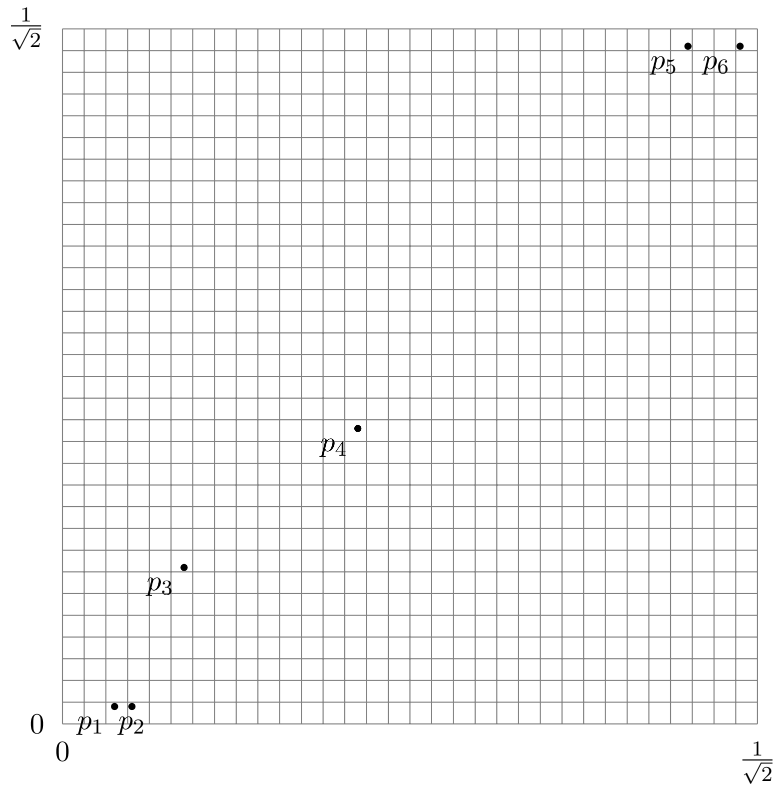

Figure 1