

## Algorithmic Game Theory

Winter Term 2021/22

Tutorial Session - Week 9

### Exercise 1:

Consider the following single-item auction: Each bidder reports a bid  $b_i \geq 0$ . The bidder with the highest bid wins the item and pays *half* his bid.

- (a) Show that if we only consider two bidders and valuations are drawn uniformly from  $[0, 1]$ , then truthful bidding is a Bayes-Nash equilibrium.
- (b) Show that this mechanism is not dominant-strategy incentive compatible.
- (c) Show that this mechanism is  $(\frac{1}{2}, 1)$ -smooth.

### Exercise 2:

Recall the auction of  $k$  identical items from the previous exercise sets. Bidder  $i$  has value  $v_i$  if he/she gets at least one of the items, 0 otherwise. We define a mechanism as follows: the bidders who reported the  $k$  highest bids win an item. Each of them has to pay their respective bids. Show that if losers (i.e. bidders who do not get any item) do not pay anything, this mechanism is  $(\frac{1}{2}, 1)$ -smooth.