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# Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 4

**Exercise 1:** (4+4 Points)

Consider the following regret-minimization-algorithm.

GREEDY

- Set $p_1^1 = 1$ and $p_j^1 = 0$ for all $j \neq 1$.
- In each round $t = 1, \ldots, T$:

  Let $L_{min}^t = \min_{i \in N} L_i^t$ for $L_i^t = \sum_{t' \leq t} \ell_i^{(t')}$ and $S^t = \{i \in N \mid L_i^t = L_{min}^t\}$.
  Set $p_i^{t+1} = 1$ for $i = \min S^t$ and $p_j^{t+1} = 0$ otherwise.

(a) Show that the costs of GREEDY are at most $N \cdot L_{min}^T + (N - 1)$.

(b) State a scheme for an example such that the stated upper bound of (a) is tight for an infinite number of values $T$.

**Exercise 2:** (5 Points)

We consider the Multiplicative-Weights Algorithm with a slightly modified update rule. Instead of using $w_i^{(t+1)} = w_i^{(t)} \cdot (1 - \eta)^{\ell_i^{(t)}}$, we now use $w_i^{(t+1)} = w_i^{(t)} \cdot \left(1 - \eta \cdot \ell_i^{(t)}\right)$. Prove a statement as in Proposition 7.7. for this modified update rule.