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Due date: November 24, 2021 at 10:00 a.m.

Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 6

Exercise 1: (2+3 Points)

Consider a second-price auction with a fixed value profile $(v_i)_{i \in N}$. Since the value profile is fixed, we get a normal-form utility-maximization game.

(a) Show that there exists a pure Nash equilibrium in the defined game.

(b) Now, consider a game in which only two players participate and $v_1 \gg v_2$ holds. Prove that even in this setting there exists a pure Nash equilibrium such that bidder 2 wins.

Exercise 2: (4+4 Points)

We consider an auction of k identical items. Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0.

(a) State a generalization of the second-price auction and prove that it is truthful (the second-price auction covers the case of $k = 1$). Follow steps in the spirit of Lecture 10.

(b) Now, consider a mechanism which sequentially performs k second-price auctions. That is, initially each bidder reports one bid. Then, in each auction, one item is sold among the remaining players using their initial bids. Show that truthful bidding does not necessarily lead to a pure Nash equilibrium even in the special case of three players and $k = 2$.

Exercise 3: (4 Points)

We consider the setting from the previous task: An auction of k identical items. Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0.

Make use of Myerson's Lemma in order to design a mechanism that is truthful. For this purpose, explicitly state the function f, verify that it is monotone, and calculate the payment rule p resulting from the integral formula.