Thomas Kesselheim
Alexander Braun

November 25, 2021
Due date: December 1, 2021 at 10:00 a.m.

Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 7

Exercise 1: (5 Points)

Recall the *Greedy-by-Value* and *Greedy-by-Sqrt-Value-Density* algorithms for single-minded CAs of lecture 12. Let us analyze another greedy algorithm that looks as follows.

Greedy-by-Value-Density

- Re-order the bids such that $\frac{b_1^*}{|S_1^*|} \geq \frac{b_2^*}{|S_2^*|} \geq \cdots \geq \frac{b_n^*}{|S_n^*|}$.
- Initialize the set of winning bidders to $W = \emptyset$.
- For $i = 1$ to n do: If $S_i^* \cap \bigcup_{j \in W} S_j^* = \emptyset$, then $W = W \cup \{i\}$.

Let $d = \max_{i \in \mathcal{N}} |S_i^*|$. Show that the given algorithm yields a d-approximation.

Exercise 2: (3 Points)

Recall the auction of k identical items from the previous exercise sets: Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0.
Make use of the VCG-results from the lecture in order to design a truthful mechanism for this auction. For this purpose, explicitly state the function f and calculate the payment rule p.

Exercise 3: (4 Points)

As seen in the lecture, let $f \colon V \to X$ be a function that maximizes declared welfare, i.e., $f(b) \in \arg\max_{x \in X} \sum_i b_i(x)$ for all $b \in V$. For each i, let h_i be an arbitrary function $b_{-i} \mapsto h_i(b_{-i})$ which does not depend on b_i. We define a mechanism $\mathcal{M} = (f, p)$ by setting

$$p_i(b) = h_i(b_{-i}) - \sum_{j \neq i} b_j(f(b)) \ .$$

Prove that $\mathcal{M}$ is a truthful mechanism.