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November 13, 2023
Due: November 20, 2023 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2023/24

Exercise Set 6

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Monday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (3 Points)

Consider the following algorithm for the Secretary Problem from Lecture 10: Do not select any of the first $\lceil \frac{n}{2} \rceil$ candidates. Afterwards, pick the first candidate whose value exceeds all previous ones. Observe that this corresponds to a threshold algorithm with $\tau = \lceil \frac{n}{2} \rceil$.
Do not use Theorem 10.1 to show that this algorithm selects the best value with probability at least $\frac{1}{4}$.

Exercise 2: (4 Points)

We consider a small variant of the Secretary Problem and the threshold algorithm. Again an adversary defines values $v_1, \ldots, v_n$. But instead of just drawing a random permutation, every candidate i draws an arrival time θ_i uniformly at random from the interval $[0, 1]$. Then the candidates arrive in increasing order of their arrival times.
The threshold algorithm now looks as follows: Observe all elements until time τ, then select the first candidate after time τ which is better than all preceding ones.
Show that for $\tau = \frac{1}{e}$ the threshold algorithm selects the best candidate with probability at least $\frac{1}{e}$.

Exercise 3: (3 Points)

Consider a rover that operates on a slope and uses solar panels to recharge. It can be in one of three states: high, medium and low on the slope.
If it spins its wheels, the probability to climb the slope (from low to medium or from medium to high or from high to high) is 0.3. With probability 0.7 it stays where it is.
If it does not spin its wheels, the probability to slide down the slope to low is 0.4. It stays where it is with probability 0.6.
Spinning its wheels uses one unit of energy per time step. Being high or medium on the slope gains three units of energy per time step via the solar panels, while being low on the slope does not gain any energy per time step. The robot wants to gain as much energy as possible in a given time horizon T.
What is the optimal policy for the rover?